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Is a truly marginal perturbation of the $G_k \times G_k$ WZNW model at $k = -2c_V(G)$ an exception to the rule?

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Abstract

It is shown that there exists a truly marginal deformation of the direct sum of two G_k WZNW models at $k = -2c_V(G)$ (where $c_V(G)$ is the eigenvalue of the quadratic Casimir operator in the adjoint representation of the group G) which does not seem to fit the Chaudhuri-Schwartz criterion for truly marginal perturbations. In addition, a continuous family of WZNW models is constructed.

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1 Introduction

The sigma-model interpretation of conformal field theories plays a crucial role in bridging string theory with point-like physics. The latter is understood as the low energy limit of the former. Crossing over this bridge leads one to discover quite fascinating features of the stringy description of space-time and forces acting in it. One of these features is that two space-times endowed with entirely different geometries and topologies could, in fact, be completely undistinguishable as they correspond to one and the same CFT. In string theory these two target spaces are described by two different conformal sigma-models which are related to each other via transformations of some coupling constants which range in a continuous interval. If at each point in this interval the conformal symmetry is preserved, then the two sigma models are said to correspond to one and the same CFT. From this point of view, the study of conformal sigma models admitting such continuous transformations is important for a better understanding string theory predictions.

The issue of continuous conformal deformations of sigma-models has attracted much of attention in the recent years (see e.g. [1]-[6]). Some of these deformations are understood as integrated infinitesimally small perturbations on a given CFT. For these perturbations to be integrable, they have to be generated by truly marginal operators, which are primary conformal operators with dimension (1,1) all the way along the perturbation. This in turn requires the renormalization group beta functions of the corresponding perturbation parameters to vanish at all values of these parameters.

Given a CFT, one can define a perturbed CFT as follows

$$S(\epsilon) = S^* - \epsilon \int d^2z \; O(z, \bar{z}), \tag{1.1}$$

where O is a perturbation operator and ϵ is some small parameter (henceforth we will be discussing only one-parameter deformations, which impose some renormalizability conditions which will be supposed to be fulfilled). The action (1.1) describes a one-parameter perturbation of a CFT given by S^*. In all cases, the operator $O(z, \bar{z})$ can be presented in the following factorized form

$$O(z, \bar{z}) = \sum_{A,\bar{A}} \lambda_{A\bar{A}} J^A(z) \bar{J}^{\bar{A}}(\bar{z}), \tag{1.2}$$

where J^A, $\bar{J}^{\bar{A}}$ are some analytic functions of z and $\bar{z}$ respectively, whereas $\lambda_{A\bar{A}}$ are some constant coefficients. J^A, $\bar{J}^{\bar{A}}$ can be called currents. However, it is unnecessary that they form any current algebra. In the case when J^A, $\bar{J}^{\bar{A}}$ do form a certain algebra, there has been suggested a criterion for the perturbation (1.1) to be truly marginal [4]. The criterion is formulated as follows: a perturbation of the type (1.2) is truly (or integrably) marginal if and only if the coefficients $\lambda_{A\bar{A}}$ are non-zero only for A, $\bar{A}$ in some abelian subalgebra of the full current algebra. Stated as above the criterion seems to provide us with both necessary and sufficient conditions. There have been some examples examined which perfectly fit into the given criterion [4].

The aim of the present paper is to exhibit one particular example of a truly marginal perturbation which does not obey the Chaudhuri-Schwartz criterion. We shall show that the direct sum of two G_k WZNW models at the special value of the level $k = -2c_V(G)$ admits a continuous perturbation by a truly marginal operator which cannot be presented as a product of abelian currents. In this way we shall prove that the CS criterion is sufficient but not necessary.

The paper is organized as follows. In section 2 we consider the action of a direct sum of two G_k WZNW models perturbed by the Thirring like current-current interaction. We show that the perturbation of the sum of two WZNW models can be recast into a perturbation of one WZNW model which appears to be quite handy for the further analysis of conformal symmetry. In section 3 we demonstrate that the perturbation operator becomes a singular vector when $k = -2c_V$. Moreover, it forms a closed fusion algebra. In section 4 we study the effect of the singular perturbation on the conformal symmetry. It turns out that the given perturbation does not break the conformal invariance. In section 5 we introduce a continuous family of WZNW models with the Wess-Zumino coupling constant $k = -2c_V$ and the sigma-model coupling constant being arbitrary. Section 6 contains our conclusion.

2 $G \times G$ WZNW model and its perturbation

By the $G \times G$ WZNW model we understand the direct product of two identical G WZNW models with level k. The action is given by

$$S_{G\times G} = S_{WZNW}(g_1, k) \ + \ S_{WZNW}(g_2, k), \tag{2.3}$$

where

$$S_{WZNW}(g_{1,2}, k) = -\frac{k}{4\pi}\left\{\int xTr|g_{1,2}^{-1}\mathrm{d}g_{1,2}|^2 + \frac{i}{3}\int \mathrm{d}^{-1}\mathrm{Tr}(g_{1,2}^{-1}\mathrm{d}g_{1,2})^3\right\}, \tag{2.4}$$

with g_1 and g_2 taking values in the Lie group G.

The theory in eq. (2.3) can be generalized in a way which does not lead to missing the underlying affine symmetries. Namely, one can add to the sum (2.3) the following interaction term [7]

$$S_I = -\frac{k^2}{\pi}\int d^2z \mathrm{Tr}^2(g_1^{-1}\partial g_1 \ S \ \bar{\partial} g_2 g_2^{-1}), \tag{2.5}$$

with the coupling S belonging to the direct product of two Lie algebras $\mathcal{G} \times \mathcal{G}$,

$$S = S_{ab} \ t^a \otimes t^b, \tag{2.6}$$

where

$$[t^a, t^b] = f_c^{ab} t^c, \tag{2.7}$$

with f_c^{ab} the structure constants of $\mathcal{G}$.

It is convenient to present the coupling matrix S_{ab} in the following form

$$S_{ab} = \sigma \hat{S}_{ab}, \tag{2.8}$$

where σ is a constant and $\hat{S}_{ab}$ is another matrix which will be thought of as being fixed. Then for small σ, the interaction term (2.5) can be observed as a perturbation of the type (1.2) on the direct sum of two WZNW models. Obviously, in this case, the currents J^A, $\bar{J}^{\bar{A}}$ are identified with the affine currents and as such form affine algebras:

$$\begin{aligned} J^A \quad &\longrightarrow \quad J_1^a = -\frac{k}{2}\mathrm{Tr}(g_1^{-1}\partial g_1 t^a), \\ \bar{J}^{\bar{A}} \quad &\longrightarrow \quad \bar{J}_2^a = -\frac{k}{2}\mathrm{Tr}(\bar{\partial} g_2 g_2^{-1} t^a). \end{aligned} \tag{2.9}$$

Correspondingly, the perturbation operator O is

$$O = \hat{S}_{ab}\ J_1^a \bar{J}_2^b, \tag{2.10}$$

and the perturbation parameter is given by

$$\epsilon = \frac{4\sigma}{\pi}. \tag{2.11}$$

Clearly, the operator O is a marginal primary operator in the $G \times G$ WZNW model. However, in the perturbed theory it may acquire anomalous conformal dimension. If it is the case, then the conformal symmetry gets broken under the perturbation. In [4], the authors calculated the two-point function of the perturbation operator in order to find the change in the conformal dimension of O. Their explicit computations went as far as to terms of order ϵ^2. We think that the analysis in [4] is incomplete and some interesting cases were missing. We are going to show that there is a more effective method of calculating perturbative effects which will allow us to prove that the results of [4] give rise to sufficient but not necessary conditions for O to be truly marginal.

We would like to start with recasting the $G \times G$ WZNW theory perturbed by the operator O into somewhat different form. Let us make the following change of variables [8]

$$\begin{aligned} g_1 &\longrightarrow \tilde{g}_1, \\ g_2 &\longrightarrow h(\tilde{g}_1) \cdot \tilde{g}_2, \end{aligned} \tag{2.12}$$

where $\tilde{g}_1,\ \tilde{g}_2$ are new variables, whereas the function $h(\tilde{g}_1)$ is the solution of the following equation

$$\partial h h^{-1} = -2k \mathrm{Tr} S \tilde{g}_1^{-1} \partial \tilde{g}_1. \tag{2.13}$$

In terms of the new variables, the action takes the form [7],[8]

$$\begin{aligned} \tilde{S}_{G\times G} &= S_{WZNW}(\tilde{g}_2, k) + S_{WZNW}(\tilde{g}_1, k) \\ &+ \frac{k^3}{\pi} \int d^2 z \mathrm{Tr} \left(\mathrm{Tr} S \tilde{g}_1^{-1} \partial \tilde{g}_1\ \mathrm{Tr} S \tilde{g}_1^{-1} \bar{\partial} \tilde{g}_1 \right) + \mathcal{O}(S^3). \end{aligned} \tag{2.14}$$

The important point to be made is that after this change of variables, the field $\tilde{g}_2$ completely decouples from $\tilde{g}_1$. As one can see $\tilde{g}_2$ is governed simply by a WZNW action, whereas the action for $\tilde{g}_1$ is more complicated. Fortunately, this action can be understood as a perturbed WZNW model [7],[8]

$$S(\tilde{g}_1) = S_{WZNW}(\tilde{g}_1, k) \;-\; \tilde{\epsilon} \int d^2z \; \tilde{O}(z, \bar{z}), \tag{2.15}$$

where

$$\tilde{O}(z, \bar{z}) = \hat{S}_{ac}\hat{S}_{bc} \; \tilde{J}^a \tilde{\bar{J}}^{\bar{b}} \tilde{\phi}^{b\bar{b}}. \tag{2.16}$$

Here

$$\begin{aligned} \tilde{J} &= -\frac{k}{2}\tilde{g}_1^{-1}\partial\tilde{g}_1, \\ \tilde{\bar{J}} &= -\frac{k}{2}\bar{\partial}\tilde{g}_1\tilde{g}_1^{-1}, \\ \\ \tilde{\phi}^{a\bar{a}} &= \mathrm{Tr}(\tilde{g}_1^{-1}t^a\tilde{g}_1 t^{\bar{a}}), \\ \tilde{\epsilon} &= \frac{4\sigma^2}{\pi}. \end{aligned} \tag{2.17}$$

Note that $\tilde{O}$ can be presented in the form (1.2), however, new currents will not form an affine algebra. Therefore, one cannot apply the CS criterion to the given operator.

In principle, one can further expand the action (2.14) in the coupling σ. However, higher order expansion is going to be much more involved as it requires a proper handling of non-local terms. In this paper we will restrict ourselves to the approximation given by eq. (2.15) which is local.

3 $k = -2c_V(G)$

In order for the perturbed theory (2.15) to be a well defined quantum field theory, the perturbation $\tilde{O}$ has to obey some consistency conditions. The crucial one is the renormalizability condition, which amounts to a condition on the OPE of the operator $\tilde{O}$ with itself [8]. This in turn imposes restrictions on the matrix $\hat{S}_{ab}$. There are a number of matrices which satisfy the given consistency condition. We will be interested in the matrix $\hat{S}_{ab}$

given by

$$\hat{S}_{ab} = \delta_{ab}. \tag{3.18}$$

It is not difficult to see that $\tilde{O}$ is a Virasoro primary operator with the conformal dimension

$$\Delta = 1 + \frac{c_V(G)}{k + c_V(G)}, \tag{3.19}$$

where the quantity $c_V(G)$ is defined as follows

$$f^{ac}_d f^{bd}_c = -c_V \ \delta^{ab}. \tag{3.20}$$

Thus, for all positive k, the operator $\tilde{O}$ is an irrelevant conformal operator from the point of view of the renormalization procedure. This circumstance would cause the infra-red divergences in the course of quantization of the perturbed theory (2.15). Therefore in general, one has to put the system into a finite box in order to avoid the IR troubles. However, there is one particular value of k at which dramatic simplifications occur.

Let us take

$$k = -2c_V, \tag{3.21}$$

where c_V is given by eq. (3.20). In this case,

$$\Delta = 0 \tag{3.22}$$

for all G. Let us now compute the norm of the given operator.

$$\begin{aligned} ||\tilde{O}||^2 &= \langle 0|\tilde{O}^\dagger(0)\tilde{O}(0)|0\rangle = \langle\tilde{\phi}^{a\bar{a}}|\tilde{J}^a_1 \tilde{\bar{J}}^{\bar{a}}_1 \tilde{J}^b_{-1} \tilde{\bar{J}}^{\bar{b}}{}_{-1}|\tilde{\phi}^{b\bar{b}}\rangle \\ &= (c_V + \frac{k}{2})^2 \langle\tilde{\phi}^{a\bar{a}}|\tilde{\phi}^{a\bar{a}}\rangle. \end{aligned} \tag{3.23}$$

We have used the standard conjugation rule of affine generators

$$\tilde{J}^\dagger_n = \tilde{J}_{-n}, \qquad \tilde{\bar{J}}{}^\dagger_n = \tilde{\bar{J}}_{-n}, \tag{3.24}$$

where

$$\tilde{J}_n = \oint \frac{dz}{2\pi i}\ z^n \tilde{J}(z), \qquad \tilde{\bar{J}}_n = \oint \frac{d\bar{z}}{2\pi i}\ \bar{z}^n \tilde{\bar{J}}(\bar{z}). \tag{3.25}$$

As one can see at $k = -2c_V$, the norm given by eq. (3.23) vanishes,

$$||\tilde{O}||^2_{k=-2c_V} = 0. \tag{3.26}$$

In other words, at the special value of k, the operator $\tilde{O}$ becomes a singular vector in the spectrum of the G-WZNW model.

It is easy to show that eq. (3.26) is, in fact, a consequence of the following equality

$$\tilde{J}^a_1 |\tilde{O}\rangle = 0, \tag{3.27}$$

which implies

$$\tilde{J}^a_{n>1} |\tilde{O}\rangle = 0. \tag{3.28}$$

Thus, as it is usually the case for singular vectors, $\tilde{O}$ is both a Virasoro primary and an affine primary. (Remember, that for $k \neq 2c_V$, $\tilde{O}$ is just a Virasoro primary.) Also one can prove that $\tilde{O}$ forms a closed algebra which results in the following relation

$$\tilde{O}(z,\bar{z})\ |\tilde{O}\rangle \sim |\tilde{O}\rangle. \tag{3.29}$$

Indeed, one can check that [11]

$$\tilde{J}^a_1 \left(\tilde{O}(z,\bar{z})|\tilde{O}\rangle\right) = 0, \tag{3.30}$$

and

$$\tilde{J}^a_0 \left(\tilde{O}(z,\bar{z})|\tilde{O}\rangle\right) = 0. \tag{3.31}$$

Hence, the right hand side of eq. (3.29) must be an affine singular primary vector which is also a singlet with respect to the global group G. The only possibility for these conditions to be satisfied is given by eq. (3.29).

4 Perturbation by the singular vector

Now we want to discuss the effect of the singular perturbation on the conformal symmetry of the perturbed theory. The latter is described by the following action

$$S(\epsilon) = S_{WZNW}(g,k)\ -\ \epsilon \int d^2z\ O(z,\bar{z}). \tag{4.32}$$

Here the operator O coincides with the operator $\tilde{O}$. From now on we will omit tilde in all expressions. The important point to be made is that the given theory is renormalizable.

This is due to the property (3.29). The renormalizability implies that the trace of the energy-momentum tensor is expressed as follows

$$\Theta = \beta(\epsilon)\, O, \tag{4.33}$$

where β is the renormalization group beta function,

$$\beta(\epsilon) = \mathrm{d}\epsilon/\mathrm{d}t. \tag{4.34}$$

We have shown in the previous section that O ($\equiv \tilde{O}$) is a singular vector. Because of eq. (4.33), the trace Θ is a singular vector as well. Hence, Θ must have no effect on the conformal symmetry of the perturbed theory. Let us show that the Virasoro central charge indeed does not change.

According to Zamolodchikov's c-theorem [9]

$$\frac{\mathrm{d}c}{\mathrm{d}t} = -12\beta^2 \langle 0|O(1)O(0)|0\rangle. \tag{4.35}$$

There holds the following relation

$$O(0)|0\rangle = J^a_{-1}\bar{J}^{\bar{a}}_{-1}|\phi^{a\bar{a}}\rangle. \tag{4.36}$$

Moreover, one can prove that

$$[J^a_{-1}, O(z,\bar{z})] = 0. \tag{4.37}$$

Then commuting J^a_{-1} in the correlator in eq. (4.35) to the left, one obtains

$$\langle 0|O(1)O(0)|0\rangle = 0 \tag{4.38}$$

and, hence,

$$\frac{\mathrm{d}c}{\mathrm{d}t} = 0. \tag{4.39}$$

Thus, c remains constant along the renormalization group flow associated with the singular perturbation, even though the beta function β is nonzero. All in all, we arrive at a conclusion that the conformal symmetry is not affected by the singular perturbation.

Now we can return to the system of two interacting WZNW models which we introduced in section 2. Obviously, since the latter system is related to the perturbation we have just discussed above, the conformal symmetry of the interacting WZNW models does

not get broken under a continuous variation of the coupling constant σ. In other words, the operator in eq. (2.10) is truly marginal. However, the parameter σ may change only in a finite interval. The restriction comes from the fact that at $\sigma = \frac{1}{4c_V}$, the system of two interacting WZNW models acquires a gauge symmetry as one can easily see by using the Polyakov-Wiegmann formula [10]. Thus, if one moves σ in the negative direction, one can continuously proceed to the value $\frac{1}{4c_V}$. Whereas in the positive direction there do not seem to occur any restrictions. It is also possible that there may be a certain duality symmetry which can mirror the restriction on the negative direction into a restriction on positive values of σ.

Now we come to our main conclusion that despite the fact that our perturbation operator does not satisfy the Chaudhuri-Schwartz criterion, nevertheless, it is a truly marginal operator. Of course, there is the issue of unitarity of the WZNW model at negative level k. However, we think that all negative normed states can be projected out by a BRST-like procedure.

5 A continuous family of WZNW models at $k = -2c_V$

In the previous sections we have exhibited that the perturbed WZNW model (4.32) arises as a lower order effective theory of two interacting WZNW models. At the same time one can consider the theory described by the action (4.32) as a fundamental quantum field theory, not as an effective model. In this case, one does not need to worry about higher order corrections in ϵ, because the action (4.32) will be the whole theory. We are going to show that this theory is, in fact, very curious.

First of all, we want to show that the perturbation O does not break the affine symmetry of the WZNW model at $k = -2c_V$. The affine symmetry is generated by the affine current $J^a(z)$. Thus, our aim is to compute the following commutator $[J^a(y), O(z,\bar{z})]$, which can be understood as equal time commutator. To this end, we present $O(z,\bar{z})$ in the factorized form

$$O(z,\bar{z}) = O(z) \cdot \bar{O}(\bar{z}), \tag{5.40}$$

where

$$O(z) =: J^a(z)\phi^a(z) : . \tag{5.41}$$

Here $\phi^a(z)$ is defined as follows

$$\phi^{a\bar{a}}(z,\bar{z}) = \phi^a(z)\cdot\bar{\phi}^{\bar{a}}(\bar{z}). \tag{5.42}$$

Note that in general the splitting into holomorphic and antiholomorphic parts can be more tricky. Namely, one can write

$$\phi^{a\bar{a}}(z,\bar{z}) = \phi^a_i(z)\cdot K^{ij}\cdot\bar{\phi}^{\bar{a}}_j(\bar{z}), \tag{5.43}$$

where K^{ij} is some matrix which does not depend on $z,\ \bar{z}$. But for our purposes, one can take $K^{ij}=1$.

We start with a definition of normal ordering in eq. (5.41). We define it according to

$$: J^a(z)\phi^a(z) :\equiv \oint\frac{d\zeta}{2\pi i}\ \frac{J^a(\zeta)\phi^a(z)}{\zeta - z}. \tag{5.44}$$

Now let us compute

$$\begin{aligned}
[J^a(y),O(z)] &= \oint\frac{d\zeta}{2\pi i}\frac{1}{\zeta - z}\left\{[J^a(y),J^b(\zeta)]\phi^b(z) + J^b(\zeta)[J^a(y),\phi^b(z)]\right\}\\
&= \oint\frac{d\zeta}{2\pi i}\frac{1}{\zeta - z}\left\{f^{ab}_c J^c(\zeta)\phi^b(z)\delta(y,\zeta) + \frac{k}{2}\phi^a(z)\delta'(y,\zeta) + f^{ab}_c J^b(\zeta)\phi^c(z)\delta(y,z)\right\}\\
&= \oint\frac{d\zeta}{2\pi i}\frac{1}{\zeta - z}\left\{\frac{f^{ab}_c f^{cb}_d}{\zeta - z}\phi^d(z)\delta(y,\zeta) + \frac{k}{2}\phi^a(z)\delta'(y,\zeta) + \frac{f^{ab}_c f^{bc}_d}{\zeta - z}\phi^d(z)\delta(y,z)\right\}\\
&+ \oint\frac{d\zeta}{2\pi i}\frac{1}{\zeta - z}[f^{ab}_c\Psi^{cb}(z)\delta(y,\zeta) + f^{ac}_b\Psi^{cb}(z)\delta(y,z)].
\end{aligned} \tag{5.45}$$

We have used the following relations

$$\begin{aligned}
\left[J^a(y),J^b(z)\right] &= f^{ab}_c J^c(z)\delta(y,z) + \frac{k}{2}\delta^{ab}\delta'(y,z),\\
\left[J^a(y),\phi^b(z)\right] &= f^{ab}_c\phi^c(z)\delta(y,z),\\
\Psi^{cb}(z) &\equiv\ : J^c(z)\phi^b(z) : .
\end{aligned} \tag{5.46}$$

By taking contour integrals in formula (5.45), we obtain

$$[J^a(y),O(z)] = (\frac{k}{2} + c_V)\phi^a(z)\delta'(y,z). \tag{5.47}$$

Therefore, at $k = -2c_V$,

$$[J^a(y), O(z)] = 0. \tag{5.48}$$

Correspondingly,

$$[J^a(y), O(z,\bar{z})] = [\bar{J}^{\bar{a}}(\bar{y}), O(z,\bar{z})] = 0. \tag{5.49}$$

Thus, the theory given by eq. (4.32) possesses both the conformal symmetry and the affine symmetry of the original WZNW model at $k = -2c_V$.

It is interesting to look at the classical limit of the operator $O(z,\bar{z})$. We find

$$O(z,\bar{z}) \longrightarrow -c_V^2 \mathrm{Tr}(\partial g \bar{\partial} g^{-1}). \tag{5.50}$$

Hence, by perturbing the WZNW model at $k = -2c_V$ by the operator (2.16), we change effectively the sigma-model coupling constant. In other words, the classical theory is described by the following action

$$S(\lambda) = \frac{1}{4\lambda} \int d^2x \ \mathrm{Tr}(\partial_\mu g \partial^\mu g^{-1}) \ - \ 2c_V \Gamma, \tag{5.51}$$

where Γ is the Wess-Zumino term. The above analysis suggests that the given theory is conformal for arbitrary (negative) coupling constant λ. This theory describes a continuous family of WZNW models with the special Wess-Zumino coupling constant $k = -2c_V$.

6 Conclusion

We have exhibited one example of a truly marginal perturbation which does not satisfy the Chaudhuri-Schwartz criterion. By using this new perturbation, we have found a continuous family of WZNW models with arbitrary sigma-model coupling constant.

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